Received: *August 11, 1999,* Accepted: *October 1, 1999*

Anomalous U(1)'s and proton stability in brane models

Luis E. Ibáñez

Departamento de Física Teórica C-XI and Instituto de Física Teórica C-XVI, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain.
E-mail: ibanez@madriz1.ft.uam.es

Fernando Quevedo

D.A.M.T.P., Silver Street, Cambridge, CB3 9EW, UK.
E-mail: F.Quevedo@damtp.cam.ac.uk

Abstract: We consider the most general generation-independent U(1) gauge symmetry consistent with the presence of Yukawa couplings for all quarks and leptons in the SUSY version of the Standard Model. This U(1) has generically mixed anomalies with SM groups, which *can not* be cancelled by the Green-Schwarz mechanism of heterotic $D = 4$ strings. We argue that these anomalies can in principle be cancelled by the generalized Green-Schwarz mechanism present in field theories corresponding to D-branes at singularities. Moreover, unlike the heterotic case, once the U(1) symmetry is broken it may remain as an exact perturbative *global* symmetry in the low energy theory. Applying this scheme to the SUSY SM we find that gauging such a general U(1): i) B and L violating operators at least up to dim $= 3, 4, 5, 6$ are generically forbidden ; ii) The μ-term is generically supressed. We also study the properties of a $U(1)_X$ symmetry whose mixed anomalies with the different SM gauge groups are in the ratio of the beta function coefficients β_a. This relation has been shown to hold in certain orientifold models. In all cases the U(1) remains as a global symmetry at the orientifold singularity, the SM Higgs can break it at the electroweak scale, making possible to relate the blowing-up of the singularity with electroweak symmetry breaking.

Keywords: D-branes, Superstring Vacua, Anomalies in Field and String Theories, Supersymmetric Effective Theories.

Contents

1. Introduction

The SUSY standard model (SM) has a number of naturality problems. One of the most pressing ones is the problem of proton stability. Indeed, the most general superpotential consistent with $SU(3)\times SU(2)\times U(1)$ gauge invariance and supersymmetry has dimension-three and -four operators which violate lepton and/or baryon number. In particular it has the general form:

$$\begin{aligned} W = &\, h_u\, Q_L u_L^c \bar{H} + h_d\, Q_L d_L^c H + h_l\, L_L e_L^c H + \\ &+ h_B\, u_L^c d_L^c d_L^c + h_L\, Q_L d_L^c L + h_L'\, L_L L_L e_L^c + \mu_L L\bar{H}\, + \mu H\bar{H} \end{aligned} \tag{1.1}$$

in a self-explanatory notation. The first line contains the usual Yukawa couplings which are needed for the standard quark and lepton masses whereas the second line shows B or L-violating couplings and the third shows the μ-terms. One needs to invoke a symmetry of some sort to forbid at least a subset of the dangerous couplings in order to obtain consistency with proton stability.

Another problematic point of the SUSY SM is the smallness of the μ-parameter, the SUSY mass of the Higgs multiplets. In principle that mass parameter would be expected to be as high as the cut-off of the theory, unless there is some symmetry reason which protects the Higgs mass from becoming large.

In field theories we are free to impose either a discrete symmetry such as R-parity or global continuous symmetries to forbid the dangerous couplings. In string theory R-parity does not in general appear as a natural symmetry and furthermore, global symmetries are believed not to be present. In fact, in perturbative string

JHEP10(1999)001

theory it can be shown [1] that there are no global symmetries (besides Peccei-Quinn symmetries of axion fields or accidental symmetries of the low-energy effective action). In nonperturbative string theory, even though there is no general proof, it is also believed that global symmetries are absent, the reason being that any theory that includes gravity will not preserve global symmetries since they are broken naturally by black holes and other similar objects.

Therefore, perhaps the simplest possibility for solving both problems in string theory would be to gauge some continuos U(1) which forbids the dangerous couplings [2, 3, 4]. This is in general problematic because, if we stick to the particle content of the MSSM such symmetries are bound to be anomalous. One might think of using the Green-Schwarz mechanism found in perturbative heterotic vacua in order to cancel those anomalies and indeed this possibility has been explored in the past. However there are two main obstructions:

1. The mixed anomalies of the U(1)'s with the SM interactions are not in the appropriate ratios to be cancelled.[1]

2. Due to the presence of a Fayet-Iliopoulos (FI) term, the U(1) symmetry is in general broken slightly below the string scale and does not survive as an exact global symmetry. Thus in general one has to rely on the particularities of the model and holomorphicity in order to supress sufficiently parameters like the μ-term [6, 7].

In the present letter we point out that these two problems are not present in the alternative generalized Green-Schwarz anomaly cancellation mechanism recently found in type I and type IIB $D = 4$, $N = 1$ string vacua. Indeed in this novel mechanism the mixed anomalies of a U(1) with the different group factors can be different. In addition, unlike what happens in the perturbative heterotic case, the FI term may be put to zero. In that case, as first pointed out in ref. [8], the U(1) survives as an effective global symmetry which is exact in perturbation theory, evading in this way the general argument against global symmetries in string theory. Both these aspects are wellcome in order to suppress the dangerous couplings with a gauged abelian symmetry.

We will discuss two general classes of flavour-independent anomalous U(1)'s. The first class allows all Yukawa interactions and it is discussed in section 3 whereas another class of anomalous U(1)'s with anomalies proportional to the beta function coefficients of the corresponding gauge groups is discussed in section 4. We will start in section 2 discussing the general aspects of anomalous U(1)'s in heterotic and type I models respectively.

[1]This can be avoided by going to generation-dependent U(1) symmetries as in ref. [5, 6]. We will concentrate in this article on flavour-independent U(1) symmetries.

2. Anomalous U(1)'s: heterotic vs type I models

In $D = 4$, $N = 1$ perturbative heterotic vacua one anomalous U(1) is often present and anomalies are cancelled by a Green-Schwarz mechanism. An important role is played by the imaginary part of the complex heterotic dilaton S, which is dual to the unique antisymmetric tensor $B_{\mu\nu}$ of perturbative heterotic strings. Under an anomalous U(1) gauge transformation $A_\mu \to A_\mu + \delta_{GS}\,\partial_\mu\theta(x)$, Im$S$ gets shifted by $\mathrm{Im}S \to \mathrm{Im}S - \delta_{GS}\,\theta(x)$, where δ_{GS} is a constant anomaly-cancelling coefficient. Since the gauge kinetic function for the gauge group G_a is at tree-level $f_a = k_a\,S$, the lagrangean contains the couplings $\mathrm{Im}S\sum_a k_a F_a \wedge F_a$, where the sum runs over all gauge groups in the model and the coefficients k_a are known as the Kac-Moody levels. Then, a shift in ImS can in principle cancel mixed U(1)-gauge anomalies. However, for this to be possible the mixed anomalies C_a have to be in the same ratios [9] as the coefficients k_a (Kac-Moody levels) of the gauge factors:

$$\frac{C_a}{C_b} \;=\; \frac{k_a}{k_b}\,. \tag{2.1}$$

With $\delta_{GS} = C_a/k_a$. In the SM, assuming the standard hypercharge normalization, we have $k_3 : k_2 : k_1 = 1 : 1 : 5/3$.

The anomalous U(1) induces, a Fayet-Iliopoulos (FI) term $\xi = g^2 M_P^2\,\delta_{GS}/16\pi^2$ at one-loop [10]. The gauge coupling g here is given by $8\pi/g^2 = \mathrm{Re}\,S$. The D-term in the lagrangean then takes the form:

$$\mathcal{L}_{\mathcal{D}} = \frac{g^2}{2}\left(\sum_i q_i X_i K_i + \xi\right)^2 , \tag{2.2}$$

where K_i is the derivative of the Kähler potential K with respect to the matter fields X_i which have charge q_i under the anomalous U(1). This term triggers gauge symmetry breaking. In order to preserve supersymmetry, the D-term has to vanish. The Fayet-Iliopoulos term ξ cannot vanish because, U(1) being anomalous implies $\delta_{GS} \neq 0$ and, in a nontrivial vacuum, $g \neq 0$. Therefore a combination of the charged fields X_i is forced to get a nonvanishing vev to compensate the FI term, breaking the anomalous U(1) and often some other non-anomalous groups.

Let us see how the anomalous U(1) gauge field gets a mass. First, the anomaly cancelling term in the lagrangian $\delta_{GS}\,B \wedge F_{\mathrm{U}(1)}$ gives rise, upon dualization, to a term proportional to $(\partial_\mu \mathrm{Im}S + \delta_{GS} A_\mu)^2$ which allows the gauge field A_μ to eat the axion ImS and get a mass, as in the standard Higgs effect. A linear combination of the real part of S and the charged scalars X_i, also gets the same mass from the Fayet-Iliopoulos term, after expanding the D term around the nontrivial vacuum. Therefore the analogy with the supersymmetric Higgs effect is complete: the original vector superfield eats the chiral superfield of the dilaton giving rise to a massive vector superfield. The anomalous U(1) symmetry gets broken at a scale determined by ξ, which can be 1-2 orders of magnitude below the Planck mass depending on the value of δ_{GS}. The massless combination of the dilaton with the charged fields X_i plays the role of the string coupling.

Since chiral fields X_i charged under the U(1) are forced to get vevs, non-renormalizable couplings of the form $\psi^n X_i^m$, where the ψ denote SM superfields will induce effective operators which will in general violate the anomalous U(1) symmetry. Thus typically this symmetry *does not survive* at low energies as a global symmetry.

Let us now see how things change in type I strings. Recently it has been realized [11] that the cancellation of U(1) anomalies in certain type I and type IIB $D = 4$, $N = 1$ models [12]–[20] proceeds in a manner quite different to the one in perturbative heterotic vacua. These are models which may be constructed as type IIB orbifolds or orientifolds [21] and contain different D-brane configurations in the vacuum. For example, the vacuum may contain a certain number of D3-branes and D7-branes with their transverse coordinates located at different positions in the extra six compact dimensions. Chiral $N = 1$ theories in $D = 4$ are only obtained when e.g., the D3-branes are located on top of orbifold singularities. It has been found [18, 11, 22] that in this class of theories: *a*) There may be more than one anomalous U(1); *b*) The mixed anomaly of the U(1) with other groups need not be universal ; *c*) There is a generalized Green-Schwarz mechanism at work in which the cancelling role is played not by the complex dilaton S but by twisted closed string massless modes M_k. These are fields which live on the fixed points of the orbifold. In particular, their real part (which are NS-NS type of fields) parametrize the smoothing out of the orbifold singularities whereas their imaginary parts (which are Ramond-Ramond fields) are the ones actually participating in the U(1) anomaly cancellation.[2] More specifically, cancellation of U(1) anomalies results from the presence in the $D = 4$, $N = 1$ effective action of the term

$$\sum_k \delta_k^l B_k \wedge F_{\mathrm{U}(1)_l} \, , \tag{2.3}$$

where k runs over the different twisted sectors of the underlying orbifold (see ref. [11, 22] for details) and B^k are the two-forms which are dual to the imaginary part of the twisted fields M_k. Here l labels the different anomalous U(1)'s and δ_k^l are model-dependent constant coefficients. In addition the gauge kinetic functions have also a

[2]More precisely, from string theory, the blowing-up modes together with the antisymmetric tensors coming from the RR sector, belong to linear multiplets. The scalar components of these multiplets, m_k are the ones that vanish at the orbifold point and their value determine the blowing-up procedure [23]. Upon dualization, the linear multiplets get switched to the chiral multiplets M_k, the relation between m_k and the real part of M_k depends on the structure of the lagrangean but close to the singularity it is linear and $m_k = \mathrm{Re}\, M_k - F(T_i, T_i^*)$ where $F(T, T^*)$ is a model dependent function which depends on the untwisted moduli fields T_i which determine the size of the compact space.

(tree-level) M_k-dependent piece[3]

$$f_\alpha = S + \sum_k s_\alpha^k M_k \,, \tag{2.4}$$

where the s_α^k are model dependent coefficients. Under a $\mathrm{U}(1)_l$ transformation the M_k fields transform non-linearly

$$\mathrm{Im} M_k \;\rightarrow \mathrm{Im} M_k + \delta_k^l \Lambda_l(x) \,. \tag{2.5}$$

This non-linear transformation combined with eq. (2.4) results in the cancellation of the $\mathrm{U}(1)_l$ anomalies as long as the coefficients C_α^l of the mixed $\mathrm{U}(1)_l$-G_α^2 anomalies are given by

$$C_\alpha^l = -\sum_k s_\alpha^k \delta_k^l \,. \tag{2.6}$$

Unlike the perturbative heterotic case, eq. (2.6) does not in general require universal mixed anomalies as in eq. (2.1).

In $D = 4$ models like these there can also be mixed $\mathrm{U}(1)_X$-gravitational anomalies. In the perturbative *heterotic* case, in order for the Green-Schwarz mechanism to work, the coefficient C_{grav}^l of those anomalies must be related to those of mixed U(1)-gauge anomalies by

$$C_{grav}^l = \frac{24}{k_\alpha} C_\alpha^l \,. \tag{2.7}$$

Such relationship disappears in the case of type IIB $D = 4$, $N = 1$ orientifolds. One can find though certain sum rules relating the gravitational to the gauge anomalies in certain classes of models. In particular, for the toroidal orientifolds of the general class studied in refs. [18] one can obtain the constraint [11]:

$$C_{grav}^l = \frac{3}{2} \sum_\alpha n_\alpha C_\alpha^l \,, \tag{2.8}$$

where n_α is the rank of the $\mathrm{U}(n)$ or $\mathrm{SO}(m)$ groups which are present in this class of orbifolds. This constraint has to be satisfied for the anomalies to be cancelled by the generalized Green-Schwarz mechanism present in these models.[4]

The FI term for this $\mathrm{U}(1)_X$ is given by $\xi = \delta_X K_M$, since the Kähler potential (to first order in M)is given [25, 26, 22, 23] by $K = [\mathrm{Re} M - F(T_i, T_i^*)]^2 + \cdots$ we can easily see that the FI term vanishes at the orbifold singularity [18, 11, 22, 23] (see

[3] This is for gauge groups coupling either to type I 9-branes or 3-branes. In the case of 5-branes or 7-branes the complex field S is to be replaced by the appropriate T_i field. The different choices for Dp-branes are in fact T-dual to each other and, hence, equivalent. See e.g. ref. [24] for details.

[4] It is amusing that eq. (2.8), valid for certain classes of type IIB orientifolds, turns out to be consistent with what is found in perturbative heterotic SO(32) *abelian* orbifolds. Indeed, in that case all mixed U(1)-gauge anomalies are equal and one has $\sum_\alpha n_\alpha C_\alpha^l = rank(\mathrm{SO}(32)) C^l = 16 C^l$. Plugging this back into eq. (2.8) we recover the perturbative heterotic result eq. (2.7).

JHEP10(1999)001

also [27, 28]). This is impossible in the heterotic case, because in that case it is the field S that cancels the anomaly and the FI-term ξ is then proportional to $K_S \sim g^2$ which cannot be set to zero in a nontrivial vacuum. The anomalous gauge field gets a mass [26, 28] exactly in the same way as in the heterotic case, nevertheless, there is no need for a charged field to get a nonvanishing vev in order to cancel the D-term, therefore the corresponding global symmetry is *not* broken. Thus, the gauge field gets a mass of the order of the string scale (since the mass depends on K_{MM^*} and not on K_M) but the global symmetry remains perturbatively exact as long as we are at the orbifold singularity $\xi = 0$. If there is a vacuum for which the D-term vanishes outside the singularity, the scale of breaking of the global symmetry could in principle be as small as we want.

We can see then that in the class of type IIB orientifolds in which this anomaly cancellation mechanism has been studied, the anomalous U(1)'s have generically a mass of order the string scale. Unlike what happens in the heterotic case, this FI-term is arbitrary at the perturbative level and hence may in principle vanish (orbifold limit). In this case the $\mathrm{U}(1)_X$ *symmetry remains as an effective global* U(1) *symmetry which is perturbatively exact.*

3. Anomalous U(1)′s and Yukawa couplings

We will now study the new possibilities offered by these generalized Green-Schwarz mechanism when applied to MSSM physics. We will consider the simplest possibility in which we extend the SM gauge group by adding a single anomalous $\mathrm{U}(1)_X$. There are just three U(1) charge asignements (beyond hypercharge) for the MSSM chiral fields which 1) allow for the presence of the usual Yukawa couplings and 2) are flavour-independent. These were named R, A and L in ref. [4], and the corresponding assignments are displayed in table 1, where we also include the hypercharge assignments Y.

Notice that L is just lepton number and R corresponds to the 3rd component of right-handed isospin in left-right symmetric models (baryon number is given by the combination $B = 6Y + 3R + 3L$). The other symmetry, A, is a Peccei-Quinn type of

	Q	u	d	L	e	H	$\bar{H}$
6Y	-1	4	-2	3	-6	3	-3
R	0	-1	1	0	1	-1	1
A	0	0	-1	-1	0	1	0
L	0	0	0	-1	1	0	0
Q_X	0	$-m$	$m-n$	$-n-p$	$m+p$	$-m+n$	m

Table 1: U(1) symmetries of the SUSY standard model.

symmetry. Thus the more general such U(1) symmetry will be a linear combination:

$$Q_X = mR + nA + pL \tag{3.1}$$

where m, n, p are real constants. We will denote the corresponding $\mathrm{U}(1)_X$'s by giving the three numbers $Q_X = (m, n, p)$. The fifth line in the table shows the general Q_X charge of the particles of the MSSM. Notice that, depending on the values of m, n, p some of the terms in the superpotential (1.1) may be forbiden. Thus, for example, $Q_X = R = (1, 0, 0)$ forbids all the terms in the second line and would forbid proton decay at this level by itself. On the other hand it does not provide an explanation for the smallness of the μ-term. In particular, the bilinear $H\bar{H}$ has Q_X charge equal to n, and hence it can only be forbiden if our U(1) has $n \neq 0$. For that purpose we can see that the U(1) symmetry is necessarily anomalous. Thus let us study the anomalies of the above Q_X symmetry.

The mixed anomalies C_i of Q_X with the SM gauge interactions are given by:[5]

$$\begin{aligned} C_3 &= -n\frac{N_g}{2}\,, \\ C_2 &= -n\frac{N_g}{2} + n\frac{N_D}{2} \;-\; p\frac{N_g}{2}\,, \\ C_1 &= -n\frac{5N_g}{6} + n\frac{N_D}{2} + p\frac{N_g}{2}\,, \end{aligned} \tag{3.2}$$

where we have preferred to leave the number of generations N_g and doublets N_D free to trace the origin of the numerical factors (one has $N_g = 3$, $N_D = 1$ in the MSSM). There is an additional constraint from the cancellation of $\mathrm{U}(1)_Y \times \mathrm{U}(1)_X^2$ anomalies which yields:

$$p(m-n) = \frac{n}{2N_g}\left(N_D(n-2m) + 2N_g m\right). \tag{3.3}$$

Thus there are only two independent parameters out of the three m, n, p if we imposse this latest constraint.

It is now easy to see that (3.2) cannot be satisfied in the *heterotic* case since there is no solution to these equations with $C_3 : C_2 : C_1 = 1 : 1 : 5/3$. However these equations can, in principle, be easily satisfied in the type I case. To see this, let us suppose for simplicity that one twisted field M is relevant in the cancellation mechanism.[6] The gauge kinetic functions for the SM interactions will have the form:

$$f_\alpha = S + s_\alpha\, M\,, \qquad \alpha = 3, 2, 1\,. \tag{3.4}$$

[5]We will not consider constraints coming from cancellation of mixed U(1)-gravitational anomalies in our analysis in this chapter and the next, since one can always cancel those by the addition of apropriate SM singlets carrying $\mathrm{U}(1)_X$ charges.

[6]Notice that M may also denote a linear combination of several twisted fields living at the singularity.

	$Qu\bar{H}$	$Qd\bar{H}$	LeH	$H\bar{H}$	$L\bar{H}$	udd	QdL	LLe
Q_X	0	0	0	n	$m-n-p$	$m-2n$	$m-2n-p$	$m-2n-p$
Q_X'	$-Y$	Y	$Y-3\delta_X$	$-\delta_X$	$-2\delta_X+Z-Y$	Z	$Z-\delta_X$	$Z-4\delta_X$

Table 2: $\mathrm{U}(1)_X$ charge of dim $=3,4$ operators in the MSSM for the two classes of U(1)'s considered in sections 3 and 4 respectively.

Now, from eq. (2.6) we see that the mixed anomalies will cancel if the anomaly coefficients in (3.2) are related to the parameters δ_X and s_α by $C_\alpha = -\delta_X \times s_\alpha$, which is possible to satisfy for appropriate δ_X and s_α. Notice , however, that in the present case the coefficients s_α are related. Indeed, for the physical case $N_g = 3, N_D = 1$ anomalies can be cancelled as long as the parameters s_α satisfy:

$$2s_3 = s_1 + s_2\,. \tag{3.5}$$

This imposes a constraint on which type I models can have an anomalous U(1) that allow all standard Yukawa couplings.

Let us now be a bit more specific and study the posibilities for anomalous $Q_X = (m,n,p)$ symmetries. As we said, we need $n \neq 0$ in order to forbid the μ-term.

1. The simplest case is obtained for $p = 0$, i.e. no gauging of lepton symmetry. In this case condition (3.3) requires $n = 2m(N_D - N_g)/N_D = -4m$ and we are left with a unique possibility Q_μ consistent with anomaly cancellation:

   $$Q_\mu = R - 4A\,. \tag{3.6}$$

   It is easy to check (see table 2) that this symmetry forbids all dimension 3 and 4 terms violating baryon or lepton number in eq. (1.1). Dangerous F-term operators of dimension smaller than 9 are also forbidden in this case (see for instance reference [29], for a recent discussion of these operators). The dimension 6 operators $[QQu^*e^*]_D$ and $[Qu^*d^*L]_D$ are however allowed (see table 3).

2. A related symmetry is the one introduced by Weinberg in 1982 in order to eliminate dangerous B, L violating operators in the supersymmetric SM [30]. In his model all quarks and leptons carry unit charge whereas the Higgses have charge -2. This symmetry corresponds to $Q_W = -5R - 4A - 6Y$. In order to cancel U(1) anomalies he was forced to add extra chiral fields transforming like $(8,1,0,-2)$+$(1,3,0,-2)$+$2(1,1,1,-2)$+$2(1,1,-1,-2)$ under $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_Y\times\mathrm{U}(1)_X$. We now see that in the present context the addition of all those extra fields is not required and one can stick to the particle content of the MSSM as long as the anomaly cancelation mechanism here considered is at work. The U(1) clearly satisfies equations (3.2) with $p = 0, n = -4$ but

JHEP10(1999)001

Operator	Dimension	Q_X Charge	Q'_X charge
$[QQQL]_F$	5	$-n-p$	0
$[uude]_F$	5	$p-n$	$-\delta_X$
$[QQQH]_F$	5	$n-m$	δ_X+Y-Z
$[\bar{H}\bar{H}e^*]_D$	5	$m-p$	$Z-2Y$
$[QuL^*]_D$	5	$p+n-m$	$2\delta_X-Z$
$[\bar{H}^*He]_D$	5	$p+n-m$	$2Y-\delta_X-Z$
$[QQd^*]_D$	5	$n-m$	δ_X-Z
$[uuuee]_F$	6	$2p-m$	$-2\delta_X-Z$
$[uddH\bar{H}]_F$	6	$m-n$	$Z-\delta_X$
$[dddLH]_F$	6	$2m-3n-p$	$Y-2\delta_X+2Z$
$[uddL\bar{H}]_F$	6	$2m-3n-p$	$2Z-2\delta_X-Y$
$[AA^*LH^*]_D$	6	$m-2n-p$	$Z-\delta_X-Y$
$[AA^*L\bar{H}]_D$	6	$m-n-p$	$Z-Y-2\delta_X$
$[QQQ\bar{H}^*]_D$	6	$-m$	$2\delta_X+Y-Z$
$[QQu^*e^*]_D$	6	$-p$	$2\delta_X$
$[Qu^*d^*H]_D$	6	$2n-m$	$Y-Z$
$[Qu^*d^*L]_D$	6	$-p$	$-\delta_X$
$[Qu^*d^*\bar{H}^*]_D$	6	$n-m$	δ_X+Y-Z
$[Qd^*d^*\bar{H}]_D$	6	$2n-m$	$-Y-Z$
$[Qd^*d^*L^*]_D$	6	$3n-2m+p$	$2\delta_X-2Z$
$[Qd^*d^*H^*]_D$	6	$n-m$	δ_X-Y-Z
$[Qu\bar{H}^*e]_D$	6	$p-m$	$Y-Z$
$[QdH^*e^*]_D$	6	$m-2n-p$	$2\delta_X-Y+Z$
$[Qd\bar{H}e^*]_D$	6	$m-n-p$	δ_X-Y+Z
$[LLH^*H^*]_D$	6	$2m-4n-2p$	$2Z-2Y-2\delta_X$
$[ddde^*]_D$	6	$2m-3n-p$	δ_X+2Z

Table 3: Supersymmetric operators of dimension 5 and 6 that violate Baryon or Lepton numbers, including their charges with respect to the anomalous U(1)'s of sections 3 and 4. Here the fields A represent any of the fields of the supersymmetric standard model.

JHEP10(1999)001

the value $m=-5$ does not satisfy the quadratic constraint (3.3). However this is exactly cancelled by the contribution from the hypercharge. Concerning what B, L-violating operators are allowed, the same as in the previos example applies.

3. For the generic case with $m, n, p \neq 0$ tables 2 and 3 show that all B, L violating operators up to dimension 6 are forbidden. A similar analysis may be done for higher dimensional operators which may be also dangerous for models with a relative small string scale [31]–[40] and [8].

4. For particular choices of m, n, p one can allow some R-parity violating dim $= 4$ operator. For example, the U(1) given by:

$$Q_{udd} = 4R + 2A + 3L \tag{3.7}$$

forbids all dim $= 3, 4, 5, 6$ lepton violating couplings but allow the coupling of type udd in the superpotential. One can also find choices which allows for lepton number violating but not for baryon number violating ones.

5. There is a particularly simple U(1) for which all the anomalies are the same: $C_1 = C_2 = C_3$. This will require a string model with identical s_α coefficients in the gauge kinetic function. This is a solution as long as $N_g = 3N_D$ which is satisfied in the physical case with $p = n/3; m = -3n/2$. Since the three s_α are identical the gauge couplings are unified for any value of $\langle \mathrm{Re}\, M \rangle$. Notice however that the $\mathrm{U}(1)_Y$ normalization is not the canonical one. This U(1) also forbids all dangerous couplings of dimension $3, 4, 5$ and 6 as well as all dangerous F-term operators of dimension less than 9.

Thus one concludes that ensuring a small μ-parameter implies in general that B and L-violating dim $= 3, 4, 5, 6$ operators are generically forbidding in the presence of anomalous U(1)'s of this type except for very particular cases. As for neutrino masses, the operator $LL\bar{H}\bar{H}$ is forbiden as long as $m \neq n + p$ so one must include right-handed neutrinos to allow for neutrino masses with charges $n + p - m$.

Now, once the $\mathrm{U}(1)_X$ symmetry is gauged, the μ parameter is forced to be zero at the perturbative level and we understand why the Higgs fields have small masses compared to the cut-off. A small but non-vanishing μ-parameter is however needed in order to obtain appropriate $\mathrm{SU}(2) \times \mathrm{U}(1)$ symmetry breaking. Notice however that once SUSY is broken, a non-vanishing D_X-term will in general appear of order M_W^2. Thus the $\mathrm{U}(1)_X$ symmetry will get small breakings of order M_W and an effective μ-term of order M_W could be generated. $\mathrm{U}(1)_X$ symmetry breaking effects could also be generated from non-perturbative effects and could also give rise to a μ term.

4. Anomalous U(1)'s and β-functions

The class of U(1) symmetries considered in the previous section is very interesting phenomenological and in principle it may be realized in type I strings. However, at the moment we do not yet know an explict example giving rise to such symmetries since we are still lacking sufficiently realistic models. We will now change our approach in the following way. Instead of imposing the phenomenological requirements of allowing all quark and lepton masses, we will impose some constraints on the anomalies inspired in some known orientifold models. Recently a special class of anomalous U(1)'s has been found in orientifold models [24, 22, 41]. These models are

	Q	u	d	L	e	H	$\bar{H}$
Q_1	1	0	0	-3	-2	0	-2
Q_2	1	0	0	-3	-2	-2	0
Q_3	0	1	0	0	-3	-1	0
Q_4	0	0	1	0	-2	-1	0
Q'_X	$m+n$	p	q	$-3(m+n)$	$-(2m+2n+3p+2q)$	$-2n-p-q$	$-2m$

Table 4: Anomalous U(1) symmetries with mixed anomalies proportional to beta function coefficients.

such that the mixed anomalies of these U(1)'s with the other gauge groups are in the ratio of the beta-function coefficients of the corresponding gauge groups. This could be of interest also in trying to accomodate a string scale well below the unification scale $M_X = 2\times 10^{16}$ GeV [24, 41]. For these U(1)'s instead of (3.1), valid for heterotic models, we would have:

$$\frac{C_\alpha}{C_\gamma} = \frac{\beta_\alpha}{\beta_\gamma}\,. \tag{4.1}$$

If an extension of the MSSM exists with such an extra $U(1)_X$, since we know the beta-function coefficients for the supersymmetric standard model, $\beta_3 : \beta_2 : \beta_1 = -3 : 1 : 11$, we can then look for the most general family independent U(1) that satisfies these constraints. Imposing the three constraints for the mixed anomalies with the standard model groups we find four independent solutions as shown in the table. The most general anomalous U(1) is

$$Q'_X = mQ_1 + nQ_2 + pQ_3 + qQ_4\,, \tag{4.2}$$

which has mixed anomalies with the standard model groups given by:

$$C_\alpha = -\frac{\beta_\alpha}{2}(2m+2n+p+q)\,. \tag{4.3}$$

We therefore will assume $2m+2n+p+q \neq 0$ so that the U(1) is indeed anomalous. These anomalies are cancelled if the gauge kinetic function has the form $f_\alpha = S + \frac{\beta_\alpha}{2}M$ (i.e. $s_\alpha = \beta_\alpha/2$) with $\delta_X = (2m+2n+p+q)$. We thus have $C_\alpha = -\frac{\beta_\alpha}{2}\delta_X$.

There are two more conditions that have to be imposed. First that the mixed $U(1)_X - U(1)_X - Y$ anomaly vanishes identically, imposing a quadratic constraint among the $U(1)_X$ charges. Second, that the $U(1)_X^3$ anomaly is also cancelled by a Green-Schwarz mechanism, therefore a constraint $C_X = -\delta_X/2\beta_x$ has also to be satisfied. The quadratic constraint can be automatically satisfied by use of the following argument: adding a term proportional to the hypercharge to each of the $U(1)_X$ charges will not change any of the linear constraints coming from the mixed $U(1)_X - G - G$ anomalies because we know that hyercharge is anomaly free. Therefore this will change only the quadratic constraint, we then assume that the proportionality constant has been fixed and not impose the quadratic constraint. This will tell us though that the four coefficients m, n, p, q are not independent and we are free to impose one constraint among them (the Weinberg U(1) of the previous section was obtained in this way). As for the cubic constraint, since it involves only the anomalous $\mathrm{U}(1)_X$ we will allow the possibility of extra matter fields charged only under the anomalous $\mathrm{U}(1)_X$ but not the Standard Model groups, which is very common in string models, so that their charges satisfy the cubic anomaly condition.

Let us then try to extract the possible implications of the $\mathrm{U}(1)_X$ symmetry. It turns out that one can draw some general conclussions without needing to go into the details of each $\mathrm{U}(1)_X$. We show in table 2 the $\mathrm{U}(1)_X$ charges of the dim $= 3, 4$ operators in the MSSM. Here one defines $Y = m - n - p$ and $Z = p + 2q$.[7] For any of those couplings to be allowed the corresponding entry has to vanish. Examining the table 1 reaches the following conclussions:

1. The μ-term is prohibited as long as the U(1) is anomalous ($\delta_X \neq 0$). This result happens to be identical to the case in the previous section. Thus this a very generic fact: $\mathrm{U}(1)'s$ forbidding the μ-term are necesarily anomalous.

2. If a mass term for the up quarks is allowed ($Y = 0$), then automatically a mass term for the down quarks is also allowed. However, at the same time, lepton masses are forbidden. Therefore with this $\mathrm{U}(1)_X$ lepton or quark mass terms may be present but not both simultaneously. This implies that the class of U(1)'s studied in the previous section does not fall into the present cathegory. In the following we will consider the case that quark masses are permitted ($Y = 0$), similar conclusions can be obtained if only the lepton masses are present, or none of them. We will comment below how leptons could get a mass.

3. If the baryon number violating operator udd is allowed ($Z = 0$) then the lepton number violating operator QdL is automatically forbidden implying, at this level, proton stability.

4. From table 3 one also observes that in the generic case all B and L-violating terms are forbidden at least up to dimension 6 except for the dim $= 5$ operator $[QQQL]_F$ which is always necesarily allowed for a U(1) of this type.

One can also check that the charges of the dimension 5 operators $LL\bar{H}\bar{H}$ are given by $2Z - 2Y - 4\delta_X$. Thus for choices with $2Z = 2Y + 4\delta_X$ neutrino masses can be naturally generated as in the standard see-saw mechanism. Alternatively, right-handed neutrinos may be added.

[7] Note that m, n, p, q are defined in eq. (4.2) and have nothing to do with those defined in eq. (3.1).

We can then see how restrictive a single anomalous symmetry can be regarding the physically interesting couplings in the superpotential. Even though the lepton masses are forbidden, there is a very economical way to generate them. If there is a standard model singlet N with charge $3\,\delta_X$, the coupling $NLeH$ is invariant under the anomalous U(1) and if N has a nonvanishing expectation value, it gives rise to lepton masses. This looks like dangerous since, as we discussed in the introduction, once we give large vevs to fields charged under the anomalous $\mathrm{U}(1)_X$, this symmetry will be broken in the effective lagrangean, as it generically hapened in the heterotic models.

Interestingly enough there is an unexpected unbroken discrete Z_3 gauge group which saves the day. Indeed, a vev for the field N turns out to break $\mathrm{U}(1)_X$ to a discrete Z_3 subgroup. This is due to the fact that the N field has to have charge $3\,\delta_X$ whereas the forbidden terms would require vevs of fields with charges $\pm 2\,\delta_X$ or $\pm 1\,\delta_X$ to be allowed. This is enough to forbid the dangerous couplings, such as the μ-term and the B, L violating operators. The ratio $\langle N\rangle/M_s$ may be at the origin of a hierarchy of fermion masses. The only dangerous coupling that is not forbidden by this $\mathrm{U}(1)_X$ (nor its residual Z_3 once leptons get masses) is the dimension 5 operator $QQQL$. We may hope that an extra symmetry, possibly a flavour-dependent U(1) or even a sigma-model symmetry as those discussed in [22], may be at work to forbid this operator and keep the proton stable. Notice that this coupling is dangerous only for the first families of quarks and leptons. A detailed analysis of flavour-dependent anomalous U(1)'s may also be interesting [5, 6, 7], in order to study the possible structure of fermion masses. We hope to report on this in a future publication.

5. Final comments

We have studied the possible use of anomalous U(1) gauge symmetries of the class found in type IIB $D = 4$, $N = 1$ orientifolds to restrict Yukawa couplings and operators in simple extensions of the MSSM in which a single such U(1) is added. We have studied in detail two general classes of flavour independent anomalous U(1)'s that may come from type I strings. The general properties of anomaly cancellation and induced FI terms are very different compared to those previously considered in the context of perturbative heterotic models. Besides its intrinsic interest, this study may lead us to extract general properties of these models. We have seen that in most cases, dangerous couplings are forbidden, in particular the μ term and B, L violating operators are naturally constrained by these anomalous symmetries. We have checked that generic symmetries of this class easily forbid B,L violating couplings up to large operator dimensions. This could be wellcome for string models with the string scale well below the Planck mass as in refs. [31]–[40] and [8].

It would be interesting to extend the present analysis to the case of flavour-dependent U(1) symmetries which could restrict the patterns of fermion mass tex-

JHEP10(1999)001

tures. Notice that in the analysis in section 3 we have tacitely assumed that the residual global $U(1)_X$ symmetry is broken (by vevs of charged scalars) only close to the weak scale so that proton decay supression is sufficiently efficient and a large μ-term is not generated. This is a possibility which is not pressent in heterotic models in which the FI-term generically forces charged scalars to get vevs close to the string scale. However in the type I context other flavour-dependent U(1)'s can be assumed to be broken by vevs of singlet scalars slightly below the string scale so that schemes analogous to those considered in [5, 6, 7] are also possible. Notice also that more than one anomalous U(1) may be present now.

If there is more than one M field at the singularity, since only one gets swallowed by the $U(1)_X$ to become massive, they can play the role of invisible axions and solve the strong CP-problem as proposed in ref. [8]. Indeed, these other fields would be massless to high accuracy and have the adequate couplings to do the job. It is unlikely that they get substantial masses after SUSY-breaking if the latter originates in a hidden sector. However it is not clear if these fields couple to $F \wedge F$. For example, in Z_3 with 9-branes, the sum of 27 fields is massive, the other 26 are not. But they have zero coupling with $F \wedge F$. We expect that in the generic case, for an anomalous $U(1)_X$, the linear combination $s_X^k M_k$ cancels the anomaly and gets eaten by the anomalous gauge field, whereas the combination $s_3^k M_k$ is the one that couples to QCD and plays the role of the QCD axion.

Once $SU(2) \times U(1)$ is broken and the Higgs fields get vevs, those vevs will also break the $U(1)_X$ symmetry. In the D_X term this can be compensated by a tiny (compared to the string scale) FI-term ξ. Thus it seems electroweak symmetry breaking will trigger a FI-term of order M_W. This means that $\langle M \rangle \propto M_W$ in these models. Thus, interestingly enough, the electroweak scale would be a measure of the blowing up of the singularity. The process of $SU(2) \times U(1)$ breaking would look like a transition of some branes to the bulk. The distance of the branes to the original singularity (given by the vevs of the Higgs) is equal to the induced FI term. Of course, all this depends on the supersymmetry breaking mechanism and how it affects the structure of the D-terms.

In the general case we can say that for an arbitrary anomalous U(1) the vacuum is either at the singularity $\xi = 0$ or not. If it is not, then the blowing-up mode can substantially affect the unification scale as argued in refs. [24, 41]. Otherwise the anomalous U(1) symmetry remains as a perturbatively exact global symmetry that can help forbidding dangerous couplings allowed by supersymmetry. This is the first concrete proposal to evade the general claim against the existence of global symmetries in string models. The SM Higgs can break this symmetry, triggering a nonvanishing value to the FI term and then moving away from the singularity. This may provide a 'brany' interpretation to the scale of electroweak symmetry breaking. In any case these new anomalous symmetries can certainly play a very interesting role in low-energy physics.

JHEP10(1999)001

Acknowledgments

This work has been partially supported by CICYT (Spain), the European Commission (grant ERBFMRX-CT96-0045), the John Simon Guggenheim foundation and PPARC.

References

[1] T. Banks and L. Dixon, *Nucl. Phys.* **B 307** (1988) 93.

[2] R. Mohapatra, *Phys. Rev.* **D 34** (1986) 3457.

[3] A. Font, L.E. Ibáñez, F. Quevedo, *Phys. Lett.* **B 217** (1989) 272.

[4] L.E. Ibáñez and G.G. Ross, *Nucl. Phys.* **B 368** (1992) 3.

[5] L.E. Ibáñez and G.G. Ross, *Phys. Lett.* **B 332** (1994) 100 [hep-ph/9403338].

[6] P. Binetruy and P. Ramond, *Phys. Lett.* **B 350** (1995) 49 [hep-ph/9412385];
P. Binetruy, S. Lavignac and P. Ramond, *Nucl. Phys.* **B 477** (1996) 353 [hep-ph/9601243];
Y. Nir, *Phys. Lett.* **B 354** (1995) 107 [hep-ph/9504312];
E. Dudas, S. Pokorski and C.A. Savoy, *Phys. Lett.* **B 356** (1995) 45 [hep-ph/9504292];
V. Jain and R. Shrock, $U(1)_A$ *models of fermion masses without a μ problem*, hep-ph/9507238;
K. Choi, E.J. Chun and H. Kim, *Phys. Lett.* **B 394** (1997) 89 [hep-ph/9611293];
P. Binetruy, N. Irges, S. Lavignac and P. Ramond, *Phys. Lett.* **B 403** (1997) 38 [hep-ph/9612442];
N. Irges, S. Lavignac and P. Ramond, *Phys. Rev.* **D 58** (1998) 035003 [hep-ph/9802334].

[7] For a review and references see: P. Ramond, *The case for a Standard Model with anomalous* U(1), hep-ph/9808488.

[8] C.P. Burgess, L.E. Ibáñez and F. Quevedo, *Phys. Lett.* **B 447** (1999) 257 [hep-ph/9810535].

[9] L.E. Ibáñez, *Phys. Lett.* **B 303** (1993) 55 [hep-ph/9205234].

[10] M. Dine, N. Seiberg and E. Witten, *Nucl. Phys.* **B 289** (1987) 589;
J. Atick, L. Dixon and A. Sen, *Nucl. Phys.* **B 292** (1987) 109;
M. Dine, I. Ichinose and N. Seiberg, *Nucl. Phys.* **B 293** (1987) 253.

[11] L. E. Ibáñez, R. Rabadán and A. Uranga, *Nucl. Phys.* **B 542** (1999) 112 [hep-th/9808139].

[12] M. Berkooz and R. G. Leigh, *Nucl. Phys.* **B 483** (1997) 187 [hep-th/9605049].

JHEP10(1999)001

[13] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Ya.S. Stanev, *Phys. Lett.* **B 385** (1996) 96 [hep-th/9606169].

[14] Z. Kakushadze, *Nucl. Phys.* **B 512** (1998) 221 [hep-th/9704059];
Z. Kakushadze and G. Shiu, *Phys. Rev.* **D 56** (1997) 3686 [hep-th/9705163];
Nucl. Phys. **B 520** (1998) 75 [hep-th/9706051].

[15] G. Zwart, *Nucl. Phys.* **B 526** (1998) 378 [hep-th/9708040].

[16] D. O'Driscoll, *General abelian orientifold models and one loop amplitudes*, hep-th/9801114.

[17] L.E. Ibáñez, *J. High Energy Phys.* **07** (1998) 002 [hep-th/9802103].

[18] G. Aldazabal, A. Font, L.E. Ibáñez and G. Violero, *Nucl. Phys.* **B 536** (1998) 29 [hep-th/9804026].

[19] Z. Kakushadze, *Phys. Lett.* **B 434** (1998) 269 [hep-th/9804110];
Z. Kakushadze, *Phys. Rev.* **D 58** (1998) 101901 [hep-th/9806044].

[20] J. Lykken, E. Poppitz and S. Trivedi, *Nucl. Phys.* **B 543** (1999) 105 [hep-th/9806080].

[21] A. Sagnotti, in Cargese 87, *Strings on orbifolds*, G. Mack et al. eds., Pergamon Press, 1988, p. 521;
P. Horava, *Nucl. Phys.* **B 327** (1989) 461; *Phys. Lett.* **B 231** (1989) 251;
J. Dai, R. Leigh and J. Polchinski, *Mod. Phys. Lett.* **A 4** (1989) 2073;
R. Leigh, *Mod. Phys. Lett.* **A 4** (1989) 2767;
G. Pradisi and A. Sagnotti, *Phys. Lett.* **B 216** (1989) 59 ;
M. Bianchi and A. Sagnotti, *Phys. Lett.* **B 247** (1990) 517 ;
E. Gimon and J. Polchinski, *Phys. Rev.* **D 54** (1996) 1667 [hep-th/9601038].

[22] L.E. Ibáñez, R. Rabadán and A. Uranga, *Sigma-model anomalies in compact type IIB orientifolds and Fayet-Iliopoulos terms*, hep-th/9905098.

[23] I. Antoniadis, C. Bachas and E. Dudas, *Gauge couplings in four-dimensional type I string orbifolds*, hep-th/9906039.

[24] L.E. Ibáñez, C. Muñoz and S. Rigolin, *Nucl. Phys.* **B 553** (1999) 43 [hep-ph/9812397].

[25] M. Douglas and G. *Moore, D-branes, quivers, and ALE instantons*, hep-th/9603167.

[26] E. Poppitz, *Nucl. Phys.* **B 542** (1999) 31 [hep-th/9810010].

[27] M. Cvetic, L. Everett, P. Langacker and J. Wang, *J. High Energy Phys.* **04** (1999) 020 [hep-th/9903051].

[28] Z. Lalak, S. Lavignac, H.-P. Nilles, *String dualities in the presence of anomalous* U(1) *symmetries*, hep-th/9903160.

[29] K. Benakli and S. Davidson, *Phys. Rev.* **D 60** (1999) 025004 [hep-ph/9810280].

[30] S. Weinberg, *Phys. Rev.* **D 26** (1982) 287.

[31] J.D. Lykken, *Phys. Rev.* **D 54** (1996) 3693 [hep-th/9603133].

[32] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315].

[33] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398];
I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quiros, *Nucl. Phys.* **B 544** (1999) 503 [hep-ph/9810410].

[34] K. Dienes, E. Dudas and T. Gherghetta, *Phys. Lett.* **B 436** (1998) 55 [hep-ph/9803466]; *Nucl. Phys.* **B 537** (1999) 47 [hep-ph/9806292]; *TeV-scale GUTs*, hep-ph/9807522.

[35] R. Sundrum, *Phys. Rev.* **D 59** (1999) 085009 [hep-ph/9805471]; *Phys. Rev.* **D 59** (1999) 085010 [hep-ph/9807348].

[36] G. Shiu and S.H. Tye, *Phys. Rev.* **D 58** (1998) 106007 [hep-th/9805157].

[37] C. Bachas, *J. High Energy Phys.* **11** (1998) 023 [hep-ph/9807415].

[38] Z. Kakushadze and S.H. Tye, *Nucl. Phys.* **B 548** (1999) 180 [hep-th/9809147].

[39] K. Benakli, *Phys. Rev.* **D 60** (1999) 104002 [hep-ph/9809582].

[40] D. Ghilencea and G.G. Ross, *Phys. Lett.* **B 442** (1998) 165 [hep-ph/9809217].

[41] L.E. Ibáñez, *Mirage gauge coupling unification*, hep-ph/9905349.

JHEP10(1999)001